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## ABSTRACT

A class of planar, multiport power dividers/combiners is presented that is a generalization of the familiar branch-line, 3-dB, quadrature hybrid. They are suitable for combining an arbitrary number of identical one-port or two-port devices while maintaining a match at the input port.

## I. Introduction

A quadrature, 3 dB hybrid has the property of input-matched combining of two identical reflection-type devices (e.g., PIN-diode phase shifters or IMPATT-diode amplifiers) without the use of circulators, as shown in Fig. 1a. Similarly, two such hybrids can be used for input-matched combining of two identical transmission-type devices (e.g., FET amplifiers) even if the devices themselves are individually mismatched, as shown in Fig. 1b. A branching cascade of these hybrids can be used to combine  $n$  devices, where  $n$  is an integer power of 2, while maintaining an input match. However, such a cascade is not suitable for an arbitrary  $n$ , and becomes cumbersome for  $n > 4$ . A class of planar,  $n$ -way power dividers/combiners, which maintains the input match for an arbitrary value of  $n$ , is presented here. These dividers/combiners are multiport generalizations of the four-port, branch-line, 3-dB quadrature hybrid.

## II. Scattering-Matrix Formulation

Consider the passive, reciprocal,  $(n+2)$ -port combining network of Fig. 2. Let  $a_i$  and  $b_i$  be, respectively, the normalized incident and reflected wave amplitudes at port  $i$ , where  $i = 1, 2, \dots, n, P, Q$ . Let a wave of unit amplitude be launched at port  $P$ , a matched load be connected to port  $Q$ , and  $n$  loads with identical reflection coefficients,  $\Gamma$ , be connected to ports  $1, 2, \dots, n$ . Thus,

$$a_P = 1; a_Q = 0; a_i = \Gamma b_i, \quad i = 1, 2, \dots, n. \quad (1)$$

For perfect combining with matched input, it is required that

$$b_P = 0, \quad b_Q = \Gamma \exp(-j\theta), \quad (2)$$

where  $\theta$  is an arbitrary phase delay.

The elements of the scattering matrices  $[S_{ij}, i, j = 1, 2, \dots, n, P, Q]$  of a class of passive, reciprocal,  $(n+2)$ -port networks satisfying (1) and (2) for all values of  $\Gamma$  are

$$S_{PP} = S_{PQ} = S_{QP} = S_{QQ} = 0, \quad (3)$$

$$S_{Pk} = S_{kP} = n^{-1/2} \exp\{-j[\theta_P + (k-1)\phi]\} \equiv p_k, \quad (4)$$

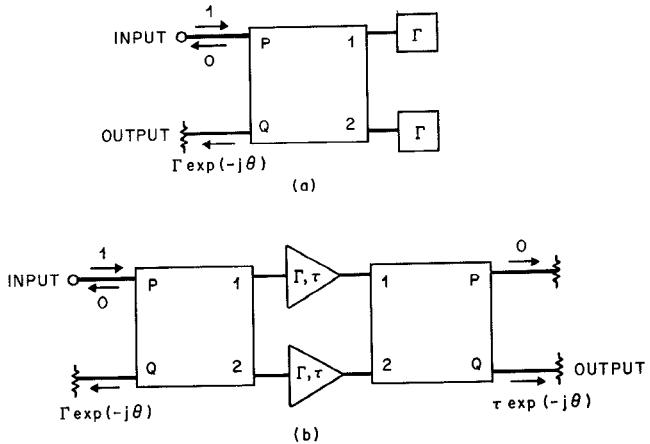


Fig. 1: The use of 3-dB quadrature hybrids for input-matched combining.

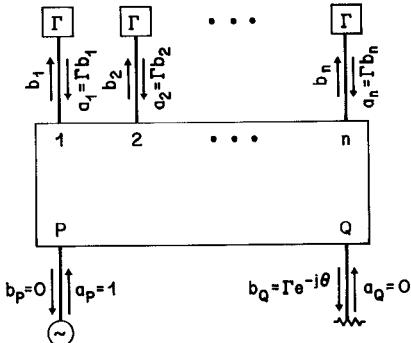


Fig. 2: A multiport generalization of the 3-dB quadrature hybrid.

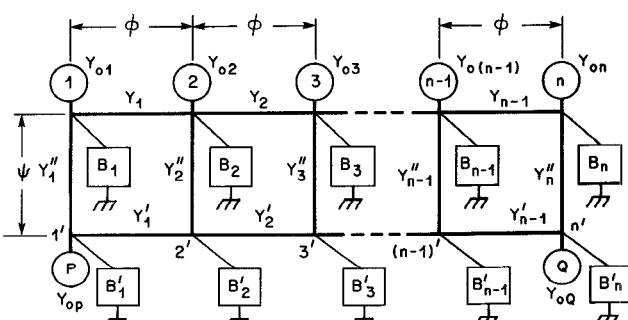


Fig. 3: Planar topology used to realize  $n(\phi)$  dividers/combiners.

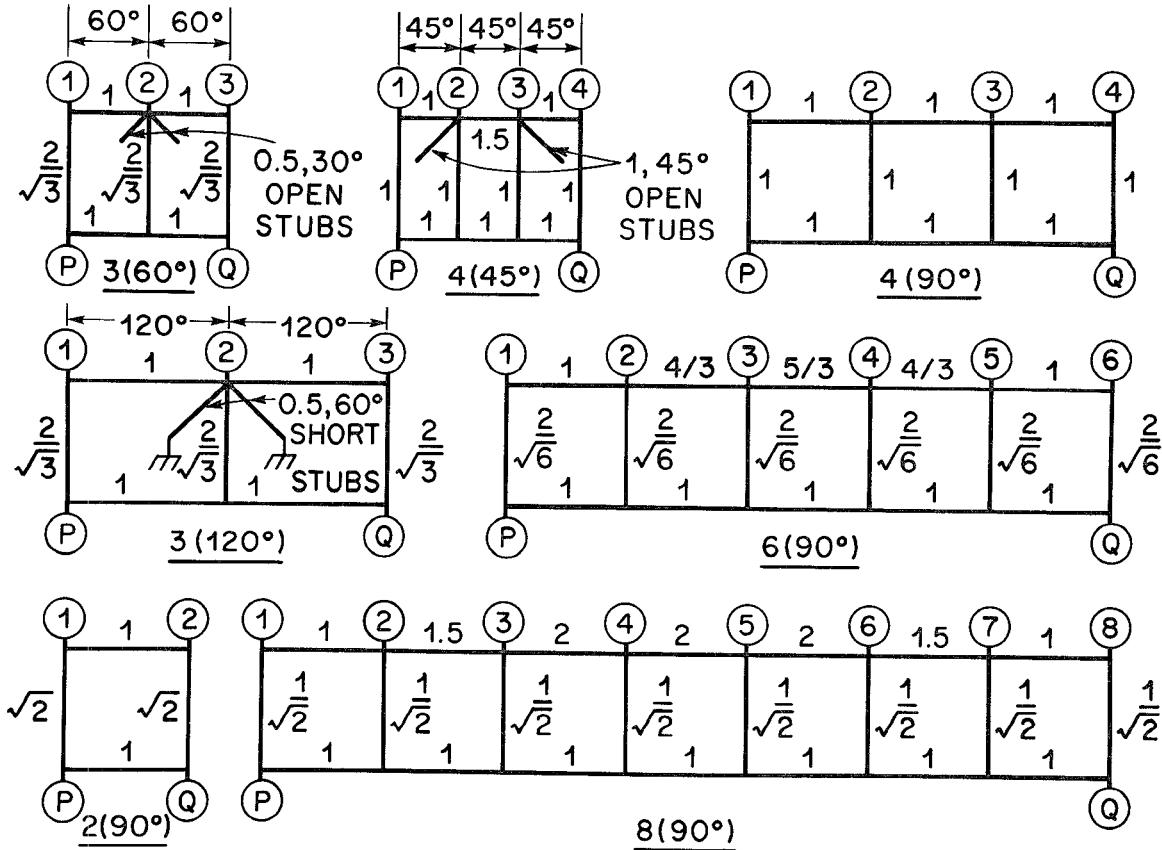


Fig. 4: Branch-line realizations of some  $n(\phi)$  dividers/combiners. All electrical lengths not shown are  $90^\circ$ . The numbers shown are characteristic admittances. All port admittances are unity.

$$S_{Qk} = S_{kQ} = n^{-1/2} \exp\left\{-j[\theta_Q + (n-k)\phi]\right\} \equiv q_k, \quad (5)$$

where  $k = 1, 2, \dots, n$ , and

$$\phi = m\pi/n, \quad m = 1, 2, \dots, \text{ or } n-1. \quad (6)$$

The symmetric  $n \times n$  matrix  $R = [S_{ij}, i, j = 1, 2, \dots, n]$  representing the interactions among ports 1 through  $n$  need not be zero (except for  $n=2$ ); rather, it satisfies

$$R_p = R_q = 0, \quad (7)$$

where  $p$  and  $q$  are the  $n \times 1$  vectors defined in (4) and (5). A network satisfying the above equations will be referred to as n(ϕ), equiampplitude, equidelay, power divider/combiner. Because of (6), the permissible combinations of  $n$  and  $\phi$  include  $2(90^\circ), 3(60^\circ), 3(120^\circ), 4(45^\circ), 4(90^\circ), 4(135^\circ), \dots$ , etc. The  $2(90^\circ)$  case is the familiar quadrature, 3 dB hybrid shown in Fig. 1. Note that two  $n(\phi)$  dividers/combiners can be employed for input-matched combining of  $n$  transmission-type devices by using an arrangement similar to that of Fig. 1b.

### III. Lossless Branch-Line Realizations

The general branch-line topology used to realize the scattering matrix defined in (3)-(7) for an  $n(\phi)$  divider/combiner is shown in Fig. 3, which is assumed to have a left-to-right symmetry. Let the

admittances of ports  $P$  and  $Q$  be equal, and let the admittances of ports 1 through  $n$  be also equal. Further, let all admittances and shunt susceptances be normalized to the admittance of ports  $P$  and  $Q$ . Thus,

$$Y_{OP} = Y_{OQ} = 1; \quad Y_{Ok} = Y_O, \quad k=1, 2, \dots, n. \quad (8)$$

With  $\phi$  given by (6), and  $\theta_P = \theta_Q = \pi/2$  in (4) and (5), it can be shown that (3)-(8) can be realized when the parameters of Fig. 3 are given by

$$\psi = \pi/2, \quad Y_\ell' = 1, \quad B_k' = 0, \quad Y_k'' = 2(Y_O/n)^{1/2}, \quad (9)$$

$$Y_\ell = Y_{n-\ell} = (Y_O/n)[\ell(n-\ell) + \sin^2(\ell\phi)/\sin^2\phi], \quad (10)$$

$$B_k = B_{n+1-k} = 2(Y_O/n)\sin[(k-1)\phi]\sin(k\phi)/\sin\phi, \quad (11)$$

where  $\ell = 1, 2, \dots, n-1$ , and  $k = 1, 2, \dots, n$ . Note that  $Y_1 = Y_{n-1} = Y_O$ , and  $B_1 = B_n = 0$ . Note also that if  $n = \text{even}$  and  $\phi = \pi/2$ , then  $B_k = 0$ .

Examples of various branch-line  $n(\phi)$  dividers/combiners are shown in Fig. 4 for  $Y_O = 1$ .

All electrical lengths not indicated on the figure are  $90^\circ$ . Note that the  $2(90^\circ)$  case is the familiar branch-line, quadrature, 3-dB hybrid [1], and that the  $4(90^\circ)$  case

uses only quarter-wave transmission lines of unity characteristic admittances!

#### IV. Interaction Among the Device Ports

Except for the 2(90°) case, the device ports, i.e., ports 1 through  $n$ , of the lossless  $n(\phi)$  dividers/combiners shown in Fig. 4 are neither individually matched nor pair-wise isolated. For example, the interaction scattering matrix  $R$ , defined in (7), for the 3(60°), 3(120°) and 4(90°) cases are given by

$$R_{3(60^\circ)} = 3^{-1} e^{\pm j2\pi/3} \begin{bmatrix} 1 & \mp 1 & 1 \\ \mp 1 & 1 & \mp 1 \\ 1 & \mp 1 & 1 \end{bmatrix}, \quad (12)$$

$$R_{3(120^\circ)} = (-j/2) \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}. \quad (13)$$

It can be shown [2] that for  $n$  ports of a network to be matched and isolated from one another, then at least  $n$  distinct resistors or terminations should be included in the network. Since ports P and Q can be counted as two terminations, at least  $n-2$  additional isolation resistors are needed if the  $n$  device ports are to be matched and isolated from one another, i.e., if  $R = 0$ . In fact, realizations can be found where exactly  $n-2$  such resistors are used. For example, a 4(90°) divider/combiner with matched and isolated device ports is shown in Fig. 5. Various possible realizations of the required isolation resistors and the associated crossovers are shown in Fig. 6.

#### V. An Experiment

A microstrip 4(90°) divider/combiner with matched and isolated device ports, Fig. 5, was built to operate at 3 GHz with 50Ω port impedances. An 0.635-mm thick alumina substrate ( $\epsilon_r \approx 10$ ) with a 50Ω/square resistive tantalum-nitride film under a chrome-gold coating was used. Each of the required 50Ω, quarter-wave, microstrip transmission lines had a width of 0.605 mm and a length of 10.2 mm, measured from center to center of the associated junctions. Each of the required 50Ω isolation resistors was made of an 0.6 mm square of the resistive film as depicted in Fig. 6a. Three ultrasonically-bonded 51×18  $\mu\text{m}$  gold ribbons were used to realize each of the crossovers. The measured frequency response is shown in Fig. 7. The operating bandwidth is in the range of 5 to 10 percent, which is typical of all the  $n(\phi)$  dividers/combiners.

#### VI. Conclusions

A new class of planar, multiport, quadrature-like, " $n(\phi)$ " power dividers/combiners has been presented. They are suitable for combining an arbitrary number of identical reflection-type or transmission-type devices while maintaining a match at the input port. They

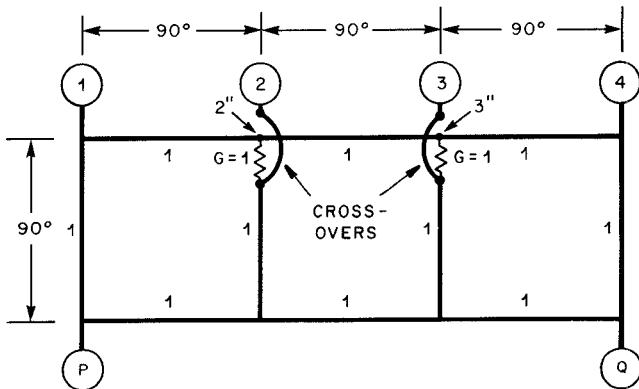


Fig. 5: A 4(90°) divider/combiner with matched and isolated device ports. All ports have  $Y_o = 1$ .

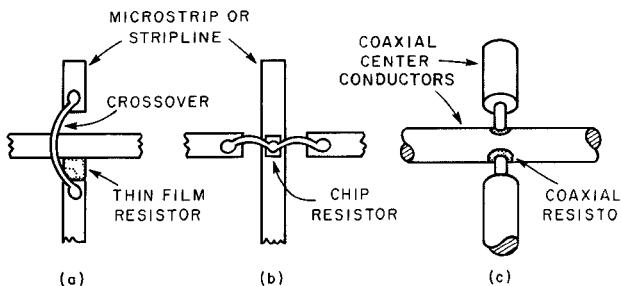


Fig. 6: Three implementations of the resistors and crossovers of Fig. 5.

can also be used for feeding multi-element antennas where progressive, equal phase delay of the signal is desired. Another interesting application is to use two lossless 3(60°) or 3(120°) dividers/combiners, one at r-f and one at i-f, to build a three-diode, filter-free, image-separation/enhancement mixer with local-oscillator isolation and noise cancellation, as shown in Fig. 8.

#### Acknowledgment

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#### References

1. C. G. Montgomery, R. H. Dicke and E. M. Purcell, Principles of Microwave Circuits, M.I.T. Rad. Lab. Series, Vol. 8, New York: McGraw-Hill, 1948, Fig. 9.37a, p. 310.
2. A. A. M. Saleh, "Theorems on Match and Isolation in Multiport Networks," IEEE Trans. Microwave Theory Tech., April, 1980.

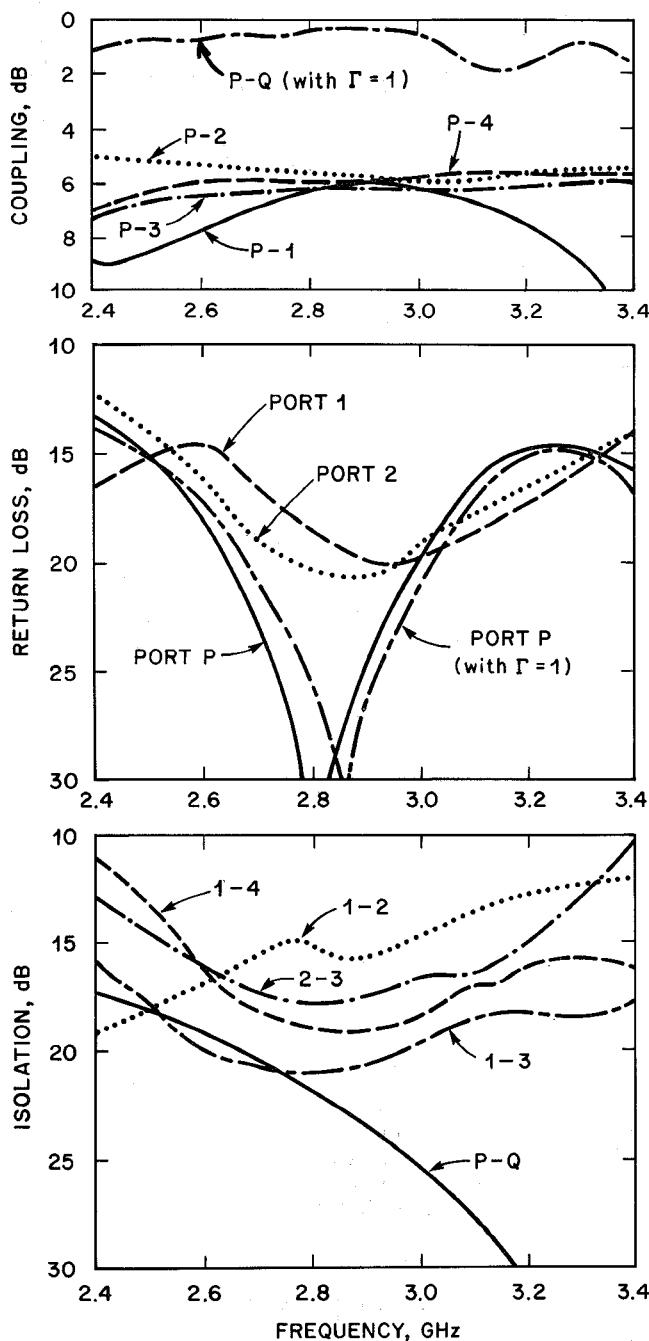


Fig. 7: Experimental results on a microstrip version of the 4(90°) divider/combiner of Fig. 5. Except for the curves labeled "P = 1", where the device ports were open circuited, all ports were terminated in matched loads.

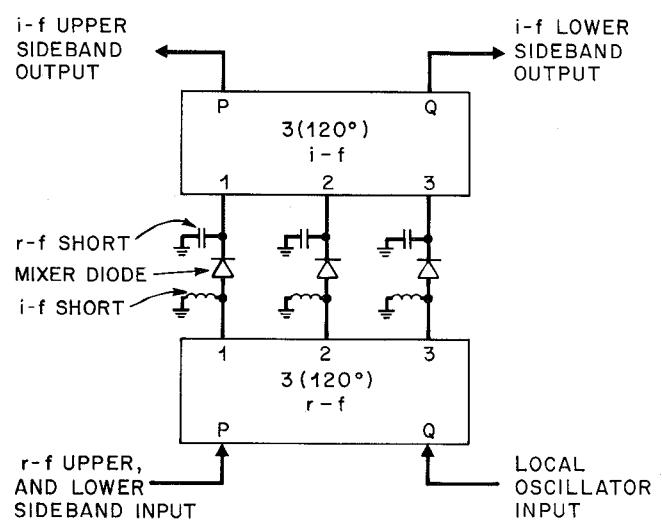


Fig. 8: A three-diode, filter-free image-separation/enhancement mixer with local-oscillator isolation and noise cancellation.